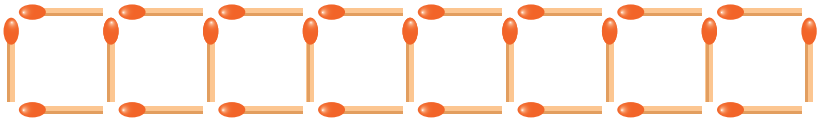
Z-kai Zoom-Up Workbook Math Grade 5

21 Let's Analyze the Meaning of Math Sentences

Date . .

Score /100

1 Look at the arrangement of matchsticks.



Alex and Benjamin found the number of matchsticks by setting up math sentences.

Alex

$1 + 3 \times 8 = 25$ (matchsticks)

Benjamin

$4 + 3 \times (8 - 1) = 25$ (matchsticks)

Interpret Alex and Benjamin's math sentences to explain how they solved the problem. (20 points eatch)

Alex [ ]

Benjamin [ ]

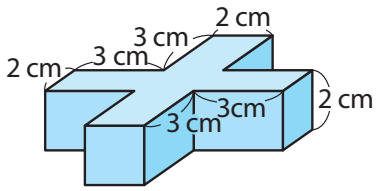
45 Challenging Problems about Volume (Part 2)

Date . .

Score /100

1 Nadine, Takeshi, and Amy tried to solve a challenging problem about the volume of a solid.

This solid looks like a "cross." What is the volume of this solid?



All of them found the solid's volume. Each person explained his/her own solution, as follows.

Nadine: I found the volume by splitting the solid into four rectangular prisms and a cube.
Takeshi: I thought about a large rectangular prism that includes this solid. Then I removed the parts that were not part of the original figure.
Amy: I looked at it as two long rectangular prisms that cross each other. I paid attention to the part where these prisms overlap.

A Which of the following math sentence represents each person's solution? (10 points each)

a. $8 \times 8 \times 2 - (3 \times 3 \times 2) \times 4$
b. $2 \times 8 \times 2 + (2 \times 3 \times 2) \times 2$
c. $(2 \times 8 \times 2) \times 2 - 2 \times 2 \times 2$
d. $(2 \times 3 \times 2) \times 4 + 2 \times 2 \times 2$

Nadine ( )

Takeshi ( )

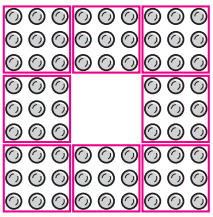
Amy ( )

21 Let's Analyze the Meaning of Math Sentences

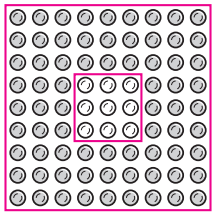
Answers

1 [Example]
Alex : He divided the shape by counting the one matchstick at the very end and then counting the remaining sets of 3 matchsticks (there are 8 sets of 3 matches which each look like a backward "C")
Benjamin : He divided the shape into a square made up of 4 matchsticks and (8 – 1) sets of 3 matchsticks.

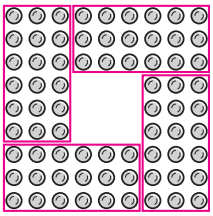
2 [Example]
A Maritza divided the shape into eight groups of 3×3 counters.



B Elle put counters in the center to fill in the shape. She used this larger group of 9 × 9 counters that contains the smaller group of 3 × 3 counters in the center, so she subtracted the 3×3 counters she added to fill in the shape.



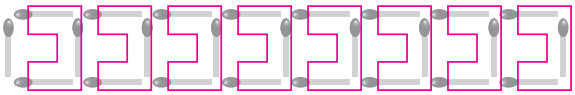
C Mario divided the shape into four groups of 6 × 3 counters.



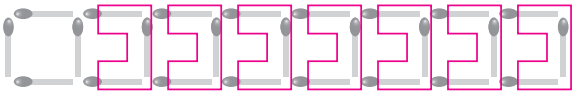
How to Think and Solve

1 There are eight groups of 3 × 3 counters that share one side. You can explain the process by drawing the figures as shown below.

(Alex)



(Benjamin)



If your response includes the points below, it is correct.

- About Alex's solution: You explained the position and number of matchsticks as one stick and (3 × 8) sticks.
- About Benjamin's solution: You explained the position and the number of matchsticks as four sticks and 3 × (8 – 1) sticks.

2 This task requires you to use math strategies to find the number of counters. Let's try to use multiple strategies and to explain simply and clearly! It's very effective to explain our reasoning by drawing and using figures.

If your responses include the two points below, you will get 10 points.

- A clear drawing (figure) that shows how the students divided the counters into square or rectangular groups.
- A clear explanation of how they divided and were thinking about the counters.

45 Challenging Problems about Volume (Part 2)

Answers

1

A Nadine: d
Takeshi: a
Amy: c

B 56 cm^3

2

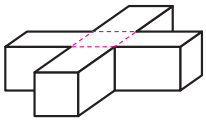
A 480 cm^3

B 456 cm^3

How to Think and Solve

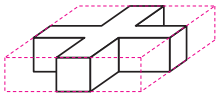
1

A In Nadine's solution, we can identify that she divided the solid along the dotted lines, as shown below.



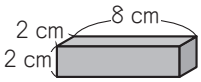
So, the math sentence, (2×3×2) × 4 + (2×2×2), represents Norimasa's solution.

In Takeshi's solution, it is assumed that he subtracted the volume of the four prisms on the corners from the volume of the larger prism drawn with dotted lines, as shown below.



So, the math sentence, (8×8×2) – (3×3×2) × 4, represents Takeshi's solution.

Amy thought about the two long rectangular prisms (as shown below) that intersect to create the cross shape.



The overlapping part where two prisms are intersecting creates a cube whose edge is 2 cm. The volume of the center cubic space is counted twice; therefore, this volume must be subtracted one time.
So, the math sentence, (2×8×2) × 2 – (2×2×2), represents Amy's solution.

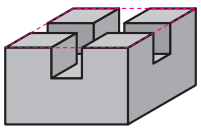
B By using Nadine's math sentence, the volume of the solid can be found by
(2×3×2) × 4 + (2×2×2) = 48 + 8 = 56 (cm^3)
Even if you use Takeshi's and Amy's math sentences, the volume will be the same.

2

A The volume of the cube is 8 × 8 × 8 = 512 (cm^3)
The hole that runs through the center is a rectangular prism with a base of 2 cm × 2 cm and a height of 8 cm.
The volume of the rectangular prism is
2 × 2 × 8 = 32 (cm^3)
So, the volume of the solid is 512 – 32 = 480 (cm^3)

B The holes are made of the two rectangular prisms whose volume was found in A above. However, these prisms are intersecting at the same height inside the cube. Therefore, the volume will be equal to the cross figure that we found in problem 1. So, the volume of this solid is 512 – 56 = 456 (cm^3).

If the top 3 cm of the solid is cut off, the figure will be shown as below.



There is another solution. Subtract the volume of two rectangular prisms whose base is 2 cm × 2 cm and height is 3 cm from the volume of solid that we found in A above.
480 – (2×2×3) × 2 = 480 – 24 = 456 (cm^3)